Central European Forum for
Migration and Population Research

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Badań Migracyjnych i Ludnościowych

Bayesian Model Selection in Forecasting International Migration: Simple Time Series Models and Their Extensions

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Plan of the presentation

1. Bayesian forecasting: general remarks
2. Bayesian model selection: introduction
3. Empirical application: Polish–German migration forecasts for 2005–2015
    - Simple stochastic processes – ARMA(1,1) sub-models
    - Models with changes in conditional variance
    - Forecasting by analogy to other migration flows
4. Robustness of forecasts against selected changes in the prior distributions
5. Concluding remarks

1. Bayesian forecasting: general remarks

Predictive distribution of forecasted values $\mathbf{x}^F$ given data $\mathbf{x}$, weighted by posterior distributions of parameters θ, equals:

$$p(\mathbf{x}^F|\mathbf{x}) = \int_\Theta p(\mathbf{x}^F|\mathbf{x}, \theta) \cdot p(\theta|\mathbf{x})\, d\theta,$$

where, by the Bayes' Theorem (1763):

$$p(\theta|\mathbf{x}) = p(\mathbf{x}|\theta) \cdot p(\theta) / p(\mathbf{x}); \quad p(\mathbf{x}) = \int_\Theta p(\mathbf{x}|\theta) \cdot p(\theta)\, d\theta$$

Posterior $p(\theta|\mathbf{x})$ is proportional to data likelihood $p(\mathbf{x}|\theta)$ and prior distribution $p(\theta)$, the latter denoting subjective expert knowledge (judgement) on the model parameters

Probability in the Bayesian statistics is also a subjective concept, interpreted as a measure of belief

2. Bayesian model selection: introduction

Theory: Uncertainty of models

- Let M_1, …, M_m be mutually exclusive (not nested) models adding up to a finite space of possible models, **M**
- Let $p(M_1)$, …, $p(M_\mathrm{m})$ be the models' prior probabilities, e.g.:
  - Flat prior (equal probabilities): $p(M_1) = \ldots = p(M_\mathrm{m})$
  - "Occam's razor" prior, favouring simpler models with smaller numbers of parameters, l_i: $p(M_i) \propto 2^{-l_i}$
- For forecasting, a model with highest posterior probability is selected on the basis of the Bayes' Theorem:

$$p(M_i|\mathbf{x}) = p(M_i)\cdot p(\mathbf{x}|M_i)\,/\,\Sigma_{k\in \mathbf{M}}\,\{p(M_k)\cdot p(\mathbf{x}|M_k)\}$$

[Hoeting et al., 1999; Osiewalski, 2001]

2. Bayesian model selection: introduction

Practice: Numerical computations

- Model Choice via Markov Chain Monte Carlo [Carlin and Chib, 1995] implemented in WinBUGS 1.4 software [Spiegelhalter et al., 2003]
- Method: iterative sampling from full conditional distributions for model-specific parameters θ_i and the model index μ:

$$\begin{cases} p(\boldsymbol{\theta}_i \mid \boldsymbol{\theta}_{j\neq i}, \mu, \mathbf{x}) \propto \begin{cases} p(\mathbf{x} \mid \boldsymbol{\theta}_i, \mu = i) \cdot p(\boldsymbol{\theta}_i \mid \mu = i) & \text{for } \mu = i \\ p(\boldsymbol{\theta}_i \mid \mu \neq i) & \text{for } \mu \neq i \end{cases} \\ p(\mu = i \mid \boldsymbol{\theta}, \mathbf{x}) = \dfrac{p(\mathbf{x} \mid \boldsymbol{\theta}_i, \mu = i) \cdot p(M_i) \cdot \prod_{j\in\mathbf{M}} p(\boldsymbol{\theta}_j \mid \mu = i)}{\sum_{k\in\mathbf{M}} [p(\mathbf{x} \mid \boldsymbol{\theta}_k, \mu = k) \cdot p(M_k) \cdot \prod_{j\in\mathbf{M}} p(\boldsymbol{\theta}_j \mid \mu = k)]}. \end{cases}$$

- Parameters θ_i are sampled either from full conditionals if μ = *i*, or from linking densities ("pseudo-priors") otherwise
- Iterations before reaching convergence are discarded

2. Bayesian model selection: introduction

Rationale for migration forecasting applications

- Features of the Bayesian approach:
  - The stochastic character ensures the formality of inference, with key focus on the uncertainty issue
  - The *a priori* expert judgement is allowed, which can supplement small-sample information (important for many time series of European migration) [cf. Willekens, 1994]
- Formal model selection techniques:
  - One way of assessing the uncertainty of model specification
  - When used with appropriate priors (e.g., "Occam's razor"), can answer the question on simplicity *versus* complexity in the population forecasting models [cf. Ahlburg, 1995; Smith, 1997]

3. Forecasts of Polish–German migration

Aim

To forecast long-term migration between Germany and Poland for 2005–2015 in different modelling frameworks

Data

- Forecasted variable – logarithms of emigration rates per 1,000 population of the sending country:

$$m_t = \ln(Mig_t / Pop_t * 1{,}000)$$

- Data series for (1985–)1991–2004
- Data sources: population stocks – Eurostat
  migration – Destatis (German data)
- Polish population stocks include post-census adjustment

3. Forecasts of Polish–German migration

a) Simple stochastic processes – ARMA(1,1) sub-models

- M_1: $m_t = c + \varepsilon_t$ [oscillations around a constant]
- M_2: $m_t = c + m_{t-1} + \varepsilon_t$ [random walk with drift]
- M_3: $m_t = c + \phi\, m_{t-1} + \varepsilon_t$; $\phi \notin \{0, 1\}$ [AR(1) process]
- M_4: $m_t = c - \theta\, \varepsilon_{t-1} + \varepsilon_t$; $\theta \neq 0$ [MA(1) process]
- M_5: $m_t = c + \phi\, m_{t-1} - \theta\, \varepsilon_{t-1} + \varepsilon_t$; $\phi,\ \theta \neq 0$ [ARMA(1,1)]

Random term: $\varepsilon_t \sim$ iiN$(0, \sigma^2)$ **Sample:** 1991–2004 (N=14)

Priors: constants $c \sim$ N$(0, 100^2)$ diffuse (hardly informative)

$\phi,\ \theta \sim$ N$(0.5, 1^2)$: processes likely stationary / time-reversible

Low precision: $\tau_{\mathrm{PL}\rightarrow\mathrm{DE}} = \sigma^{-2} \sim \Gamma(0.25, 0.25)$; $\tau_{\mathrm{DE}\rightarrow\mathrm{PL}} \sim \Gamma(4, 0.4)$

3. Forecasts of Polish–German migration

b) AR(1) extensions: non-constant conditional variance

General model: $m_t = c + \phi\, m_{t-1} + \varepsilon_t$, where $\varepsilon_t \sim \mathrm{N}(0, \boldsymbol{\sigma_t^2})$

- M_5: $\sigma_t^2 = \sigma^2$ [reference model, constant variance]
- M_6: $\sigma_t^2 = k + \alpha \cdot \varepsilon_{t-1}^2$ [AR(1)–ARCH(1) process]
- M_7: $\sigma_t^2 = k + \alpha \cdot \varepsilon_{t-1}^2 + \beta \cdot \sigma_{t-1}^2$ [AR(1)–GARCH(1,1)]
- M_8: $\ln(\sigma_t^2) = k + \gamma \cdot \ln(\sigma_{t-1}^2) + \zeta_t$ [simple stochastic volatility, SV]

Deterministic (M_6–M_7) vs. stochastic (M_8) changes in variance

2nd random term: $\zeta_t \sim \mathrm{iiN}(0, \rho^2)$ **Sample:** 1985–2004 (N=20)

Priors: c, ϕ as before, other concentrated for computational reasons: $\alpha, \beta, \gamma \sim \Gamma(10, 20)$; $k \sim \Gamma(1, 0.1)$; $1/\rho^2 \sim \Gamma(10, 1)$

3. Forecasts of Polish–German migration

c) Models with analogy to Iberian migration flows

Idea: to capture institutional changes, e.g., post-accession opening of the Western EU labour markets [Kupiszewski, 1998]

- M_{10}: $m_t = c + \varepsilon_t$ [reference model, no analogy]
- M_{11}: $m_t = c + a \cdot m^{PT}{}_{t-18} + b \cdot \mathbf{1}_{t=2002} + \varepsilon_t$ [Portugal]
- M_{12}: $m_t = c + a \cdot m^{ES}{}_{t-18} + \varepsilon_t$ [Spain]
- M_{13}: $m_t = c + a \cdot m^{IB}{}_{t-18} + b \cdot \mathbf{1}_{t=2002} + \varepsilon_t$ [both countries]

Rationale: timing of EU accession, system transformation

Random term: $\varepsilon_t \sim$ AR(1) **Sample:** 1992–2004 (N=13)

Priors: c, ϕ, τ as before, $a \sim$ N(0.5, 1^2) – a positive analogy

3. Forecasts of Polish–German migration

Bayesian model selection for three proposed classes M

Model posteriors $p(M_i|\mathbf{x})$ under "Occam's razor" priors, $p(M_i) \propto 2^{-l_i}$

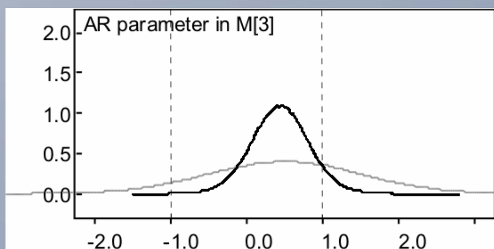
Migration flow	Subclasses of ARMA(1,1)					Extensions of variance				Models with analogy			
	M_1	M_2	M_3	M_4	M_5	M_6	M_7	M_8	M_9	M_{10}	M_{11}	M_{12}	M_{13}
Poland → Germany	**0.42**	0.29	0.14	0.11	0.03	0.12	0.10	0.05	**0.74**	**0.69**	0.07	0.14	0.10
Germany → Poland	0.23	**0.49**	0.16	0.08	0.03	**0.71**	0.02	0.00	0.27	**0.88**	0.01	0.09	0.02
Model priors $p(M_i)$	0.31	0.31	0.15	0.15	0.08	0.50	0.25	0.13	0.13	0.40	0.20	0.20	0.20

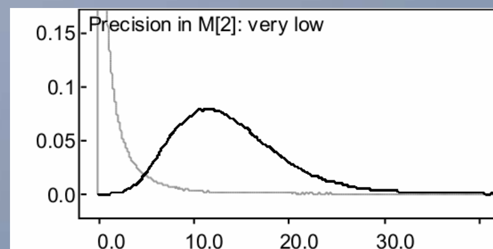
- Simple random models (oscillations / random walks)
- Either constant or stochastic conditional variance
- No linear analogies supported by the data

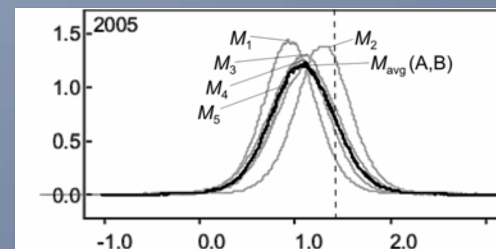
3. Forecasts of Polish–German migration

Overview of selected empirical results

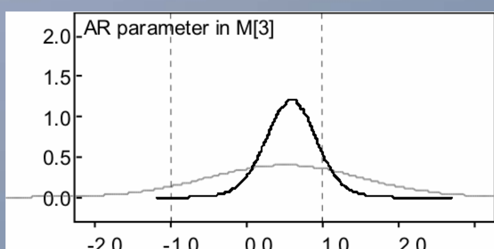
Migration from Poland to Germany: selected distributions, forecast for 2005

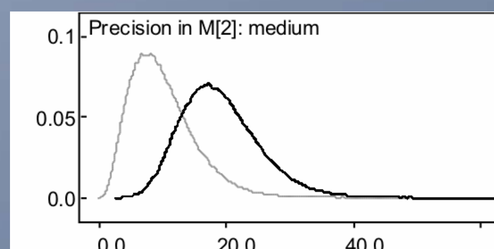


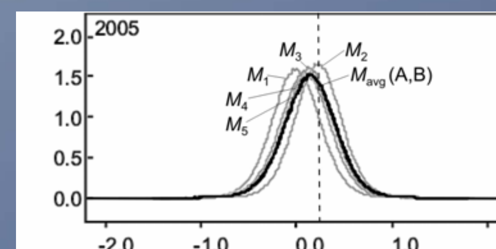




Migration from Germany to Poland : selected distributions, forecast for 2005







Grey lines – distributions *a priori*, black – *a posteriori*.

Highest *ex post* errors for M_1, lowest – for M_2

3. Forecasts of Polish–German migration

Exp(m_t) forecasts from the selected models, 2005–2015

Model	Forecasted exp(m_{2005})			Forecasted exp(m_{2010})			Forecasted exp(m_{2015})		
	10%	Median	90%	10%	Median	90%	10%	Median	90%
Poland → Germany: exp(m_{2004}) = **3.65;** exp(m_{2005}) = **4.17**									
M_1: oscillation	**1.77**	**2.57**	**3.72**	**1.78**	**2.57**	**3.71**	**1.78**	**2.57**	**3.72**
M_9: AR(1)–SV	**2.64**	**3.42**	**4.41**	**1.74**	**3.31**	**7.18**	**1.60**	**3.28**	**8.40**
M_{10}: no analogy	**1.96**	**2.99**	**4.41**	**1.71**	**2.62**	**4.04**	**1.71**	**2.63**	**4.05**
Germany → Poland: exp(m_{2004}) = **1.27 ;** exp(m_{2005}) = **1.28**									
M_2: RWD	**0.91**	**1.25**	**1.71**	**0.48**	**1.18**	**2.90**	**0.28**	**1.12**	**4.36**
M_6: AR(1)	**0.90**	**1.26**	**1.76**	**0.69**	**1.24**	**2.36**	**0.63**	**1.24**	**2.71**
M_9: AR(1)–SV	*0.94*	*1.25*	*1.67*	*0.72*	*1.21*	*2.24*	*0.68*	*1.21*	*2.62*
M_{10}: no analogy	**0.80**	**1.12**	**1.55**	**0.69**	**1.01**	**1.48**	**0.69**	**1.01**	**1.48**

- Median trajectories plausible, indicate stabilisation
- Limits of 80-percent predictive intervals reasonable, except for the (likely) non-stationary models (RWD/AR)

4. Robustness against changes in priors

a) Uniform model priors $p(M_i)$ instead of the "Occam's razor"

- Results for ARMA(1,1) sub-models: the same ones selected, although with different posterior probabilities $p(M_i|\mathbf{x})$

b) Alternative prior distributions for θ, used as a reference: the non-informative ones [Jeffreys, 1961]

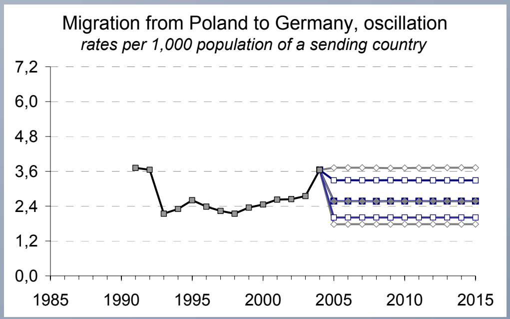
- In practical applications, "hardly informative" priors can be used for computational convenience: [Congdon, 2003]

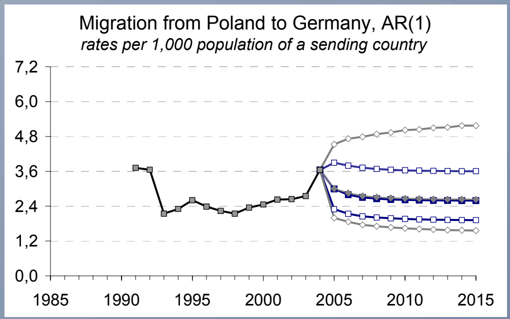
  For structural parameters: N(0, D^2), where D is a big number

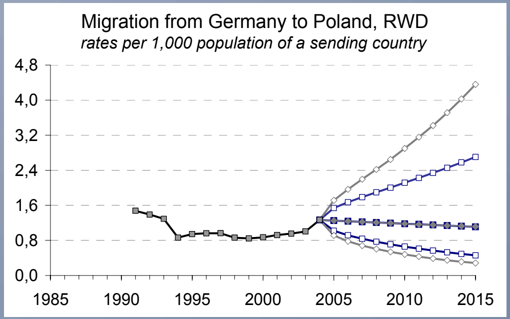
  For precision $\tau = \sigma^{-2}$: $\Gamma(a, a)$, with small parameters a
- Under D = 100 and a = 0.001, the obtained forecasts for oscillations, random walks and AR(1) differ from the "informative" ones with respect to uncertainty estimates

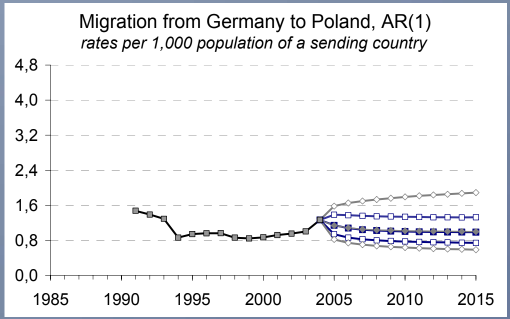
4. Robustness against changes in priors

Results: informative *vs.* hardly informative priors on precision









Without *a priori* assumptions on low precision τ, in many cases the 80-percent intervals are narrower than within-sample variability

5. Concluding remarks

- Bayesian model selection techniques allow for identifying models with the highest data support and for assessing uncertainty on various levels, including model specification
- Empirical results: simple, unstructured models preferred
- Selection of oscillations, random walks and stochastic volatility models confirms a hardly predictable character of both migration rates and their uncertainty measures
- Given the shortness of data series, the results are not robust against changes in priors (especially for precision)
- However, without assuming low precision *a priori*, the predictive intervals would be in many cases too narrow as for such an uncertain phenomenon as migration

Thank you for your attention!

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